Radial Thickness Calculation and Visualization for Volumetric Layers

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Abstract

Volumetric layers often encountered in medical image analysis are characterized by double and nested bounding surfaces. The thickness of a volumetric layer at a point on the bounding surface is the distance from that point to the opposite surface. There exist several definitions for the layer thickness. A newly proposed thickness definition is the radial thickness, which is defined as the distance between each pair of corresponding points on the two surfaces with the same polar coordinate. The thickness values calculated by the radial thickness definition are unique and does not depend on the starting surface. In this paper, we describe a class for calculating the radial thickness of one volumetric layer represented as coupled and nested triangle meshes.

The class, `vtkRadialThicknessCalculate`, is implemented using the Visualization Toolkit (VTK), `www.vtk.org`. In this document, we describe the radial thickness calculation algorithm and provide the user with the source code and the input data to reproduce the results. The radial thickness calculation described in this paper has a variety of applications including the thickness calculation for the skull vault, which is the original motivation for this work.

Contents

1 Introduction

Volumetric layers encountered in medical image analysis usually contains double and nested bounding surfaces, for instance, the skull vault [5], the cerebral cortex [6], and the myocardium of the left ventricle [1]. In the recent work [6], we described a generic automatic landmarking method for structures with coupled surfaces by minimizing the description length. In that method, the local thickness gradients are considered in formulating the description length. Once the landmark on one surface is determined, its counterpart on the other surface can be found directly. It has been empirically shown that considering the thickness information in landmarking volumetric layers will lead to models with higher quality compared with performing the landmark optimization on two surfaces independently. In that work, we proposed a new thickness measurement that is suitable to measure the thickness of the volumetric layers. In this paper, we will give the detailed implementation of this thickness definition as well as the usage of the appended source code.

The thickness of a volumetric layer at a point on the bounding surface is the distance from that point to the opposite surface. There exist several definitions for the layer thickness. The most popular ones are the closest thickness (T_{close}) and the normal thickness (T_{normal}) [2]. T_{close} is the distance from a point on one surface to the closest point on the other. T_{normal} is the distance from a point on one surface to the point on the other in the direction of the surface normal. To find a generic measure that performs reasonably on every type of layers is impractical. We determine the layer thickness as the distance between each pair of corresponding points on the two surfaces with the same polar coordinate. This measure is named as the radial thickness (T_{radial}). We illustrate the measures of T_{close}, T_{normal}, and T_{radial} on an axial plane of the skull boundary (see Figure 1). Different from measures T_{close} and T_{normal} that depend on the starting surface, the T_{radial} is unique and landmarks are grouped in pairs using this measurement.

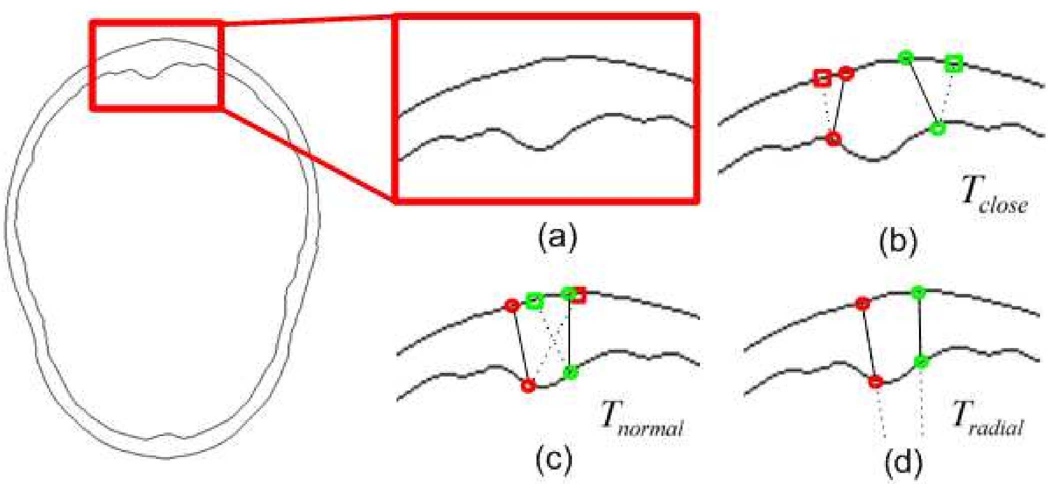


Figure 1: Different thickness definitions illustrated on a part of one axial plane of the skull boundary: (a) the coupled surfaces; (b) the closest thickness measure; (c) the normal thickness measure; (d) the radial thickness measure used in this paper.

2 Algorithm Description

To calculate the radial thickness, the two coupled triangle meshes are treated as a master mesh and a supplementary one. As illustrated in Figure 1 (d), each ray is determined by the vector from the center to each vertex in the master mesh. The radial thickness calculation is to determine the distance between each vertex on the master mesh and the intersection point of the corresponding ray with the supplementary surface. Specifically, this problem can be reduced to the problem of checking if there is any intersection of a ray and a triangle in the supplementary mesh, and determining its coordinates if it exists. We adopt the fast and minimum-storage algorithm of ray/triangle intersection examining proposed in [3]. A ray emitted from the center C_0 to one vertex V in the master mesh is defined as

$$R(t) = C_0 + tD, \tag{1}$$

where $D = (V - C_0)/\|V - C_0\|$ is the normalized direction. A point, $T(u,v)$, on a triangle defined by three vertices V_0, V_1, V_2 is represented by

$$T(u,v) = (1-u-v)V_0 + uV_1 + vV_2, \tag{2}$$

where (u,v) are the barycentric coordinates, which satisfies that u and v are non-negative, as well as the sum of u and v is not greater than one. It is apparent that the intersection between the ray $R(t)$ and the triangle $T(u,v)$, is equivalent to $R(t) = T(u,v)$, which leads to the following equation

$$C_0 + tD = (1-u-v)V_0 + uV_1 + vV_2. \tag{3}$$

By rearranging the terms in Eqn.(3), we can easily have,

$$[-D, V_1 - V_0, V_2 - V_0] \begin{bmatrix} t \\ u \\ v \end{bmatrix} = O - V_0. \tag{4}$$

By using the Cramer's rule and defining $E_1 = V_1 - V_0$, $E_2 = V_2 - V_0$, and $T = O - V_0$, we obtain the solution to Eqn.(4) as

$$\begin{bmatrix} t \\ u \\ v \end{bmatrix} = \frac{1}{[-D, E_1, E_2]} \begin{bmatrix} |T, E_1, E_2| \\ |-D, T, E_2| \\ |-D, E_1, T| \end{bmatrix}. \tag{5}$$

Considering that

$$\begin{aligned} C_1, C_2, C_3 &= -(C_1 \times C_3) \cdot C_2 && (6) \\ &= -(C_3 \times C_2) \cdot C_1, && (7) \end{aligned}$$

we can further simplify Eqn.(5) to speed up the computations in the following form

$$\begin{bmatrix} t \\ u \\ v \end{bmatrix} = \frac{1}{[P \cdot E_1]} \begin{bmatrix} Q \cdot E_2 \\ P \cdot T \\ Q \cdot D \end{bmatrix}, \tag{8}$$

where $P = (D \times E_2)$ and $Q = T \times E_1$.

3 User's Guide

The `vtkRadialThicknessCalculate` class takes one VTK mesh input as the master mesh and the other VTK mesh input as the supplementary mesh. The output is the radial thickness values, or the normalized ones between 0 and 1. Each radial direction is determined by the vector from the center to each vertex in the master mesh. There is a distance value for every direction. If there is no intersection between the ray and the supplementary mesh, the radial thickness value will be set to 0. Alternatively, the user can also specify the directions in a text file, in which each row with 3 real numbers represent a direction.

Before calculating the radial thickness, we need to include the header file

```
#include "vtkRadialThicknessCalculate.h",
```

and declare an instance in the following way,

```
vtkRadialThicknessCalculate radialThicknessCal;
```

The master mesh and the supplementary mesh can be set by

```
radialThicknessCal.SetMasterMesh(inner); \\

// 'inner' is an inner skull mesh in the format of VTK
```

and

```
radialThicknessCal.SetSupplementaryMesh(outer); \\

// 'outer' is an outer skull mesh in the format of VTK
```

Users can specify the file name to save the radial thickness values by

```
radialThicknessCal.SetThicknessFileName("thicknessFile");
```

Actually there will be two output files. One is "thicknessFile.txt", which contains the radial thickness values, while the other output is "thicknessFile_normalized.txt", which saves the normalized ones.

Finally, the thickness calculation can be triggered by

```
radialThicknessCal.StartThicknessCalculate();
```

Aside from using the vector from the center to each vertex in the master mesh, which is the default mode, the user can also explicitly specify the directions by setting a direction file in the following way

```
radialThicknessCal.SetDirectionsFileName("directions");
```

4 Results

In this section, we firstly describe the packages required to generate the results presented in this study. The radial thickness calculation and visualization is performed on the human skull vaults afterwards.

4.1 Required Packages

In order to reproduce the results presented in this paper, the user needs to compile and run the attached code with the following packages:

- CMake 2.4.6
- Visualization Toolkit VTK 5.0.3
  The class `vtkRadialThicknessCalculate` is based on the Visualization Toolkit. The output can be a file containing the normalized radial distances according to the vertex order in the master mesh, or according to the direction order if the direction file is specified. The distance values can be plotted on the master mesh to visualize the distance variation explicitly.
- KWMeshvisu [4]
  KWMeshvisu is adopted to visualize the calculated radial distances on the master mesh. Each radial thickness for the corresponding vertex in the master mesh is color-coded for visualization. This kind of thickness representation provides a qualitative understanding of the layer data, such as the regions with high and low thickness values. The normalized thickness file is in the right form as required by KWMeshvisu, which looks as follows,

```
NUMBER_OF_POINTS = 5685
DIMENSION = 1
TYPE = Scalar
```

- Neurolib [1]
  The VTK2Meta tool in the MetaMeshTools project from the Neurolib package is adopted to convert the mesh data from the VTK format to the Meta format, which can be loaded in KWMeshvisu for visualization.

4.2 Results on Human Skull Vaults

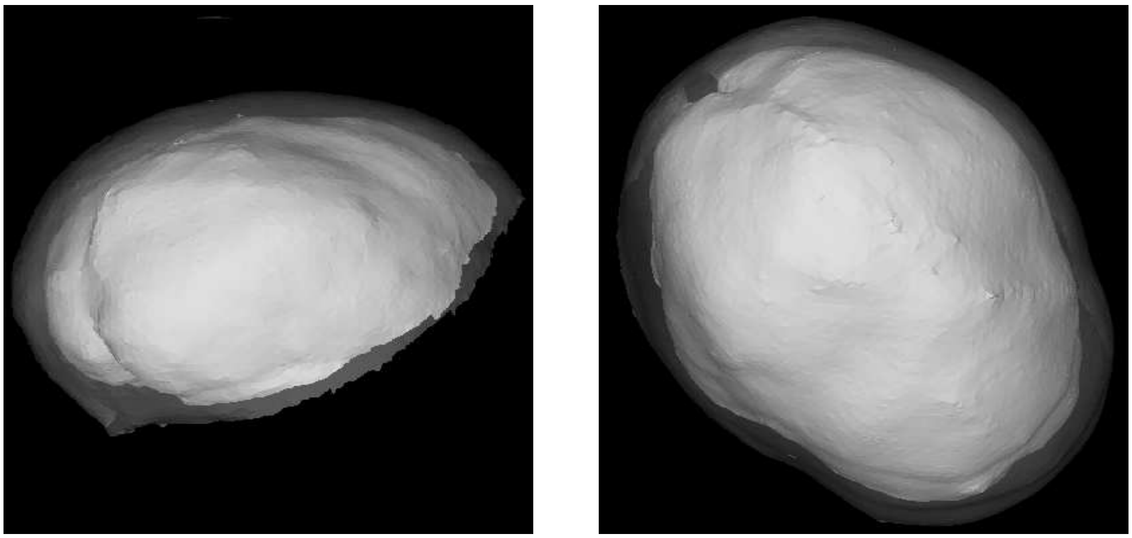
To illustrate the applicability of the proposed class, we apply it to calculate the local thickness of the human skull vault, which is defined as the upper part of the skull and is an open coupled-surface structure. The skull volume was segmented from the head CT data acquired at the Prince of Wales Hospital, Hong Kong. The field of view of the CT data is 512×512 and the voxel size is $0.49\text{mm} \times 0.49\text{mm} \times 0.63\text{mm}$.

Figure 2 shows the coupled (inner and outer) surfaces of the skull vault viewed from the back and the top respectively. The radial thickness values are calculated using the algorithm described in this paper. Figure 3 and 4 present the color-coded thickness values on the inner and outer surfaces of the skull vault, respectively. In Figure 3, the inner surface is taken to be the master surface, while the outer surface is the selected to be the master surface in Figure 4. Note that in our program, if there is no intersection between the ray and the

[1] `http://www.ia.unc.edu/dev/`

supplementary surface, we artificially set the thickness value at that direction to be zero, which causes the appearance of a ribbon of zero values in the inner surface of the skull vault as shown in Figure 3(a).

From the results, we can find that the resultant radial thickness values represent the layer thickness reasonably, which is consistent with our observation in Figure 2. Thus, it can be concluded that for the structures with a near-spherical morphology radial thickness is a suitable thickness measurement. Another observation is no matter whether the master surface is the inner surface or the outer surface, the resultant thickness value for each particular vertex will not be altered, except in the marginal regions.



(a) (b)

Figure 2: Coupled surfaces of the skull vault: (a) the back view; (b) the top view.

5 Conclusion

In this paper, we provide the implementation of a new thickness definition for the volumetric layer structures proposed in [6]. This implementation is encapsulated in the `vtkRadialThicknessCalculate` class, which also contains a realization of the fast ray/triangle intersection algorithm [3]. The provided class is potentially applicable to various problems involving the assessment of the thickness of the volumetric layer structures.

6 Acknowledgements

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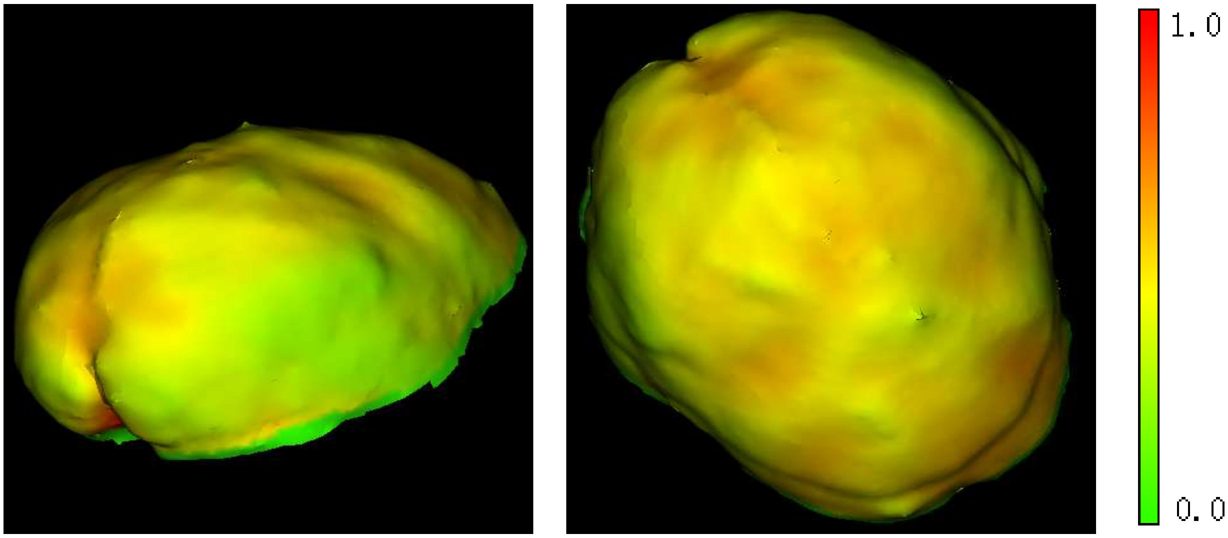
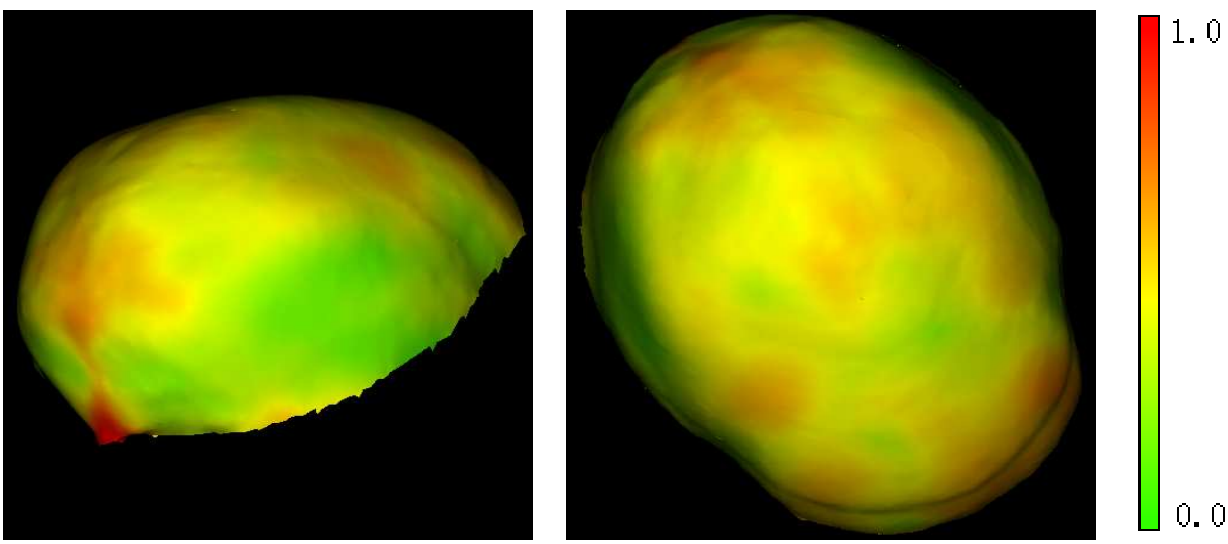
A Implementation Notes

The `vtkRadialThicknessCalculate` class takes two VTK mesh files as inputs. One is the supplementary mesh file, and the other is the master mesh. The center and the radial directions of the volumetric layer can



(a) (b) (c)

Figure 3: The color-coded thickness values plotted on the surface of the inner skull: (a) the back view; (b) the top view; (c) the color bar used to code the normalized thickness values.



(a) (b) (c)

Figure 4: The color-coded thickness values plotted on the surface of the outer skull: (a) the back view; (b) the top view; (c) the color bar used to code the normalized thickness values.

be calculated from the master mesh data. In both the user-specified mode and the default mode, for each direction, the distance from the center to the supplementary mesh is calculated as the distance between the center and the intersection between the ray and a triangle in the supplementary mesh. In the user-specified mode, the distance to the master mesh can be calculated in the same way as to the supplementary mesh. Whereas, in the default mode, the distance from the center to the master mesh in a certain direction can be achieved directly as the distance between the center and the vertex in that direction. Besides using the distance files as outputs, there exist other two APIs to access distances directly:

```
vtkFloatArray* GetRadialThickness();
```

```
vtkFloatArray* GetNormalizedRadialThickness();
```

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