# A Mix-resolution Bone-related Statistical Deformable Model (mBr-SDM) for Soft Tissue Prediction in Orthognathic Surgery Planning

Qizhen He[1,2], Horace H. S. Ip[1] and James Xia[3]

May 21, 2008

[1]Image Computing Group，Department of Computer Science, City University of Hong Kong, Hong Kong, China
[2] Department of Computer Science and Technology, University of Science & Technology of China, Hefei, China
[3]Surgical Planning Laboratory, Department of Oral and Maxillofacial Surgery,The Methodist Hospital Research Institute, U.S.A

**Abstract**

In this paper, we propose a Mix-resolution Bone-related Statistical Deformable Model (mBr-SDM) to improve the predicting accuracy of orthognathic surgery, particularly for the main deformation region. Mix-resolution Br-SDM consists of two separate Br-SDM of different resolutions: a high-resolution Br-SDM which is trained with more samples to capture the detail deforming variations in the main deforming regions of interest, together with a low-resolution Br-SDM which is trained with a smaller number of samples to capture the major variations of the remaining facial points. The experiments have shown that the mix-resolution Br-SDM is able to significantly reduce the predicting error compared with the corresponding Finite Element Model, while giving a low computational cost which is characteristic of the SDM approach.

## Contents

# 1 Introduction

Orthognathic surgery aims to correct for abnormities of the facial anatomies. Computer aided planning of such surgeries has been an area of active research in the past decades because the predicted facial outcome of the surgery helps surgeons to choose the best surgical strategy among the possible surgical plans, as well as to improve the communications between the surgeons and the patients. Conventionally, the appropriate osteotomy line and the necessary displacements of the jaw segment are determined by 2D cephalometry. The advent of high quality medical imaging modalities (such as CT images) has made possible accurate and efficient representation and prediction of the 3D facial changes as a result of surgery, and at the same time, posed many new challenging problems, among them is the prediction of facial soft tissue deformation as a result of craniofacial bone movements. The Mass Spring Models (MSM) has been introduced [1] to model the facial tissue as masses and springs connecting neighboring masses. The model structure is intuitive, and computational cost of predicting tissue deformation is low. The major disadvantage is that the parameters in a MSM, such as the spring constant [4], typically do not bear direct relation to the biomechanical properties of human soft tissues. Later on, Finite Element Models (FEM) [2], as a general discretization procedure of continuum problems is suggested to solve the problem of facial deformation. FEM is accurate [3], but computational and memory intensive which makes it not particularly suitable for real-time surgical planning where interactive response with the user is a key requirement. While the Mass Tensor Model (MTM) [5] provides a model that has the simplicity of MSM as well as the accuracy of FEM, the computational demand for prediction process using high resolution models is still far from real-time responses. Statistical Deformable Model (SDM) which has been developed originally for object segmentation [6] and motion analysis [7] has been introduced for soft tissue prediction by Meller in 2005[8]. With SDM, the system is able to learn the prior knowledge of tissue deformation from a set of training samples, and predict facial changes according to the learned knowledge. This method, while it is efficient, it suffers from the small sample size problem [9] which is typical of many other applications of SDM. This problem is particularly significant in surgical planning applications because typically we do not have many real life instances of medical organ samples. Additionally, in [8], the authors used the pre-operational facial model to predict the post-operational facial changes by assuming that all patients underwent the same standard surgery, and, more important, the approach does not take bone movements into account. Thus it is not particularly applicable to orthognathic surgical planning where different surgical plans would be investigated and evaluated.

To harness the accuracy of FEM and the computational efficiency of SDM as well as taking into account the bone-movement that cause the tissue deformation in the first place, we have introduced a novel statistical deformable model called Bone-related SDM or ***Br-SDM*** in [10]. In Br-SDM, FEM is first applied to generate a large sample set of soft-tissue deformation instances with respect to different jaw-bone movements, then the generated set of deformation samples are used to train a Statistical Deformable Model (SDM) for subsequent surgical planning, which is eventually used to predict the facial changes for specific jaw movements. The experimental results demonstrate that the Br-SDM has comparable accuracies with FEM (the average predicting difference of the two methods stayed within 10% of the jaw movement) while using only 10% of the computational time and memory of conventional FEM. However, it is also observed that, the predicting differences between Br-SDM and FEM in the main deforming area, e.g. the region of the chin, has the highest errors compared with the other points on the face by almost 20% to 30% of the jaw movement. One possible cause of this phenomenon may be due to the insufficient sample size for the training of the SDM as well as insufficient resolution of the bone and soft tissue meshes around those facial regions.

To address the above problems, the primary contribution of this paper is that we propose a novel mixed-resolution Br-SDM (mBr-SDM) which consists of a high-resolution SDM, called *sub-SDM*, for the main deforming regions of interest which is trained with more samples to capture the detail deforming variations in the main deforming area, together with a low-resolution SDM, called *main-SDM*, which is trained with a smaller number of samples to capture the variations of the remaining facial points. The experiments have shown that the sub-SDM is able to reduce the predicting error compared with FEM significantly, while the maintaining the low computational cost which is characteristic of our original Br-SDM approach. The resulting SDM is called ***Mixed-resolution Br-SDM*** because it consists of two separate SDMs each with a different mesh resolution and different training sample size which enables precise prediction of soft tissue deformation as a result of bone movement, particularly for the facial areas where the main deformation occurs.

The rest of this paper is organized as follows. Section 2 briefly summarizes the work described in [10]. Section 3 presents Mixed-resolution Br-SDM, with the experimental results shown in section 4. We conclude our paper in section 5.

## 2 Formulation of Br-SDM

We have previously proposed a Bone-related SDM or Br-SDM to achieve both accurate and efficient prediction for orthognathic surgery planning. For the detail formulation of the construction of a Br-SDM, we refer to [10]. We give a brief summary of the technique in the following.

Using conventional Finite Element Method (FEM), we can generate different facial outlook according to different surgical plans. Then for each output, displacements of the boundary points (which reflect the jaw movements of the plans) and the displacements of the non-boundary points (which reflect the facial appearance as predicted by FEM) form a sample $X = (\delta_{boundary}, \delta_{non-boundary})^T = (\delta_1, ..., \delta_m, \delta_{m+1}, ..., \delta_n)^T$, where $\delta_i$ is the displacement of vertex $i$ of on the soft-tissue mesh, with the first $m$ vertices overlapped with the jaw mesh and defined as *boundary points*, and the remaining *n-m* points which are free to deform and defined as *non-boundary points*.

All these samples are collected and used in the construction of a Statistical Deformable Model (SDM):

$$X = \overline{X} + \Phi b \quad \textbf{(1)}$$

where $\overline{X}$ is the mean of the sample, calculated by $\overline{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$ with the sample count $N$, and $\Phi = (p_1, ... p_t)$ is the major variation modes with the eigen-vectors $p_1, ... p_t$ correspond to the largest $t$ eigen-values of the covariance matrix $S$ calculated by $S = \frac{1}{N}\sum_{i=1}^{N} dX_i dX_i^{\ T}$ .

Given a statistical deformable model for bone-related soft tissue prediction, a surgery plan can be expressed in terms of the cutting plane of the jaw model and the displacement of the jawbone pieces. These displacements can be further transformed into $\delta_{boundary}$ to predict $\delta_{non-boundary}$ . Then by minimizing

$$D(b) = \left\| \delta_{boundary} - (\overline{\delta}_{boundary} + \Phi_{boundary} b) \right\|^2 \qquad (2)$$

where $\overline{\delta}_{boundary}$ and $\Phi_{boundary}$ are the non-boundary part of $\overline{X}$ and $\Phi$ respectively, we can choose appropriate variation parameter $b$ to fit $\delta_{boundary}$ into Br-SDM. And then $b$ is used further to calculate $\delta_{non-boundary}$ which represent the facial changes by:

$$\delta_{non-boundary} = \overline{\delta}_{non-boundary} + \Phi_{non-boundary} b \qquad (3)$$

where $\overline{\delta}_{non-boundary}$ and $\Phi_{non-boundary}$ are the non-boundary part of $\overline{X}$ and $\Phi$ respectively.

# 3 A Mixed-resolution Br-SDM (mBr-SDM)

## 3.1 Motivation

We have shown through experiments that while the Br-SDM presented above can achieve a good average accuracy for post-operative prediction of soft tissue deformation which is around 10% of the predicted deformation by FEM [10], we also observed that the predictive errors for certain areas of face are higher than that for the other areas. Specifically, when we visualize the prediction errors of each point on the soft-tissue mesh according to their positions, we find that the major differences lie in the two sides around the chin, as illustrated in Fig.1.

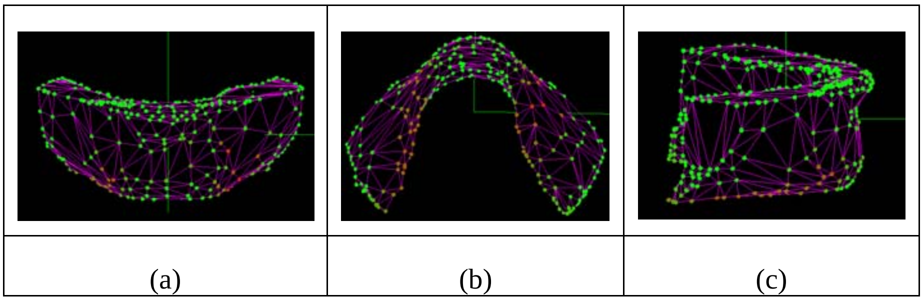



**Fig. 1.** The differences (colour-coded) in predicted movement of the vertices (comparing with FEM) for the surgical plan of moving jaw-bone forward 5.1mm. (a) the front view, (b) the bottom view and (c) the side view. Green points: points with the least difference (0 mm), red points: points with the largest difference (2.3 mm), the other points: the color is interpolated between red and green according to the difference.

From Fig.1 and fig 2, we can see that points lying on both sides of the chin have the largest differences in terms of predicted movements between Br-SDM and FEM. In Fig.2, we can see that the points within the main deform area suffer from relatively large prediction differences of up to 2.3 mm, while the other points proved to have good accuracy within differences of below 0.5mm. Unfortunately, it is the area, which suffers large prediction differences are the place where we are most interested in, and we need therefore to improve the prediction accuracy compared with FEM.

The reason for large difference may be that points in this area have more deforming variation modes. To capture the large number of fine deformation modes, we need a higher-resolution mesh model and more training samples. But we also need to make sure that the computational requirement does not increase significantly at the same time. To this end, we introduce a mixed-resolution Br-SDM which consists of a high-resolution Statistical Deformable Model (called *sub-SDM*) for the region of particular interests while keeping the original low-resolution SDM (called *main-SDM*) to model the deformations in other areas.

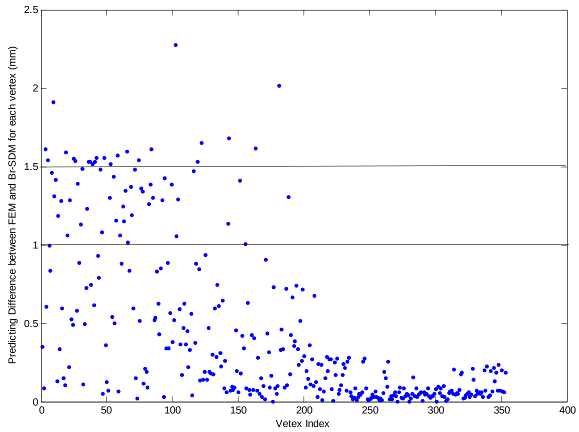
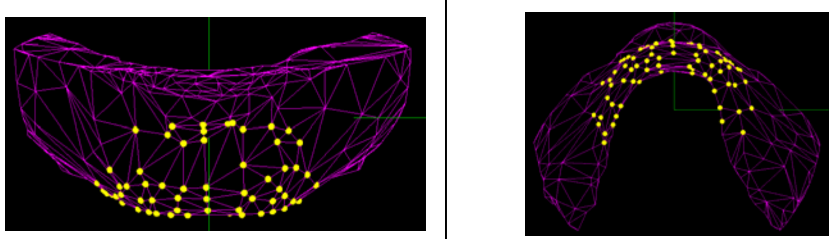



**Fig. 2.** Differences in the predicted movements between FEM and Br-SDM of each vertex for the surgical plan of moving jaw-bone forward 5.1mm.

## 3.2 The structure of the Mixed-resolution Br-SDM

The points on the soft-tissue mesh consists of two sets, one of which, we call set A, consists the points within the main deformation area of interest, e.g. the area of the chin as mentioned before (illustrated in Fig.3), and the other set, called set B, consists the other points.



(a) (b)

**Fig. 3.** Points within the main deformation area (marked as yellow). (a) the front view and (b) the bottom view

Consequently, the sample defined in section 2 is divided into two parts, $X_A = \{\delta_{boundary}, \delta_{A-non-bundary}\}$ and $X_B = \{\delta_{boundary}, \delta_{B-non-bundary}\}$, where $\delta_{A-non-bundary}$ are the displacements of the points in set A, and $\delta_{B-non-bundary}$ are the displacements of the points in set B.

Then as described in [10], we use FEM to generate samples, and these samples are transformed into $\{X_A\}$ and $\{X_B\}$ respectively. $\{X_B\}$ is used to train the ***main-SDM*** :

$$X_B = \overline{X}_B + \Phi_B b \tag{4}$$

with $\overline{X}_B = \{\overline{\delta}_{boundary} \quad \overline{\delta}_{B-non-boundary}\}$ the mean of $\{X_B\}$ and $\Phi_B = \begin{pmatrix} \Phi_{B-boundary} \\ \Phi_{B-non-boundary} \end{pmatrix}$ the variation mode.

As mentioned before, we needs more samples to capture the variation modes within $\{X_A\}$. So introduce and simulate more surgical plans to generate more samples in this critical facial area, and these samples are incorporated into $\{X_A\}$, to train the ***sub-SDM*** :

$$X_A = \overline{X}_A + \Phi_A b \tag{5}$$

with $\overline{X}_A = \{\overline{\delta}_{boundary} \quad \overline{\delta}_{A-non-boundary}\}$ the mean of $\{X_A\}$ and $\Phi_A = \begin{pmatrix} \Phi_{A-boundary} \\ \Phi_{A-non-boundary} \end{pmatrix}$ the variation mode.

To predict the soft tissue changes of a given surgical plan $\delta_{boundary}$, the displacements of the points in set A $\delta_{A-non-boundary}$ are found using (5), by minimizing

$$D_{X-A}(b_A) = \left\| \delta_{boundary} - (\overline{\delta}_{boundary} + \Phi_{A-boundary} b_A) \right\|^2 \tag{6}$$

and calculating

$$\delta_{A-non-boundary} = \overline{\delta}_{A-non-boundary} + \Phi_{A-non-boundary} b_A \tag{7}$$

where $\Phi_{A-boundary}$ and $\Phi_{A-non-boundary}$ are the first and second parts of $\Phi_A$ corresponding to the boundary and non-boundary points respectively, $\overline{\delta}_{boundary}$ and $\overline{\delta}_{A-non-boundary}$ are the first and second parts of $\overline{X}_A$ corresponding to the boundary and non-boundary points respectively, and $b_A$ is the variation parameter estimated in (6) and taken into (7) to compute $\delta_{A-non-boundary}$.

Similarly, the displacements of the points in set B $\delta_{B-non-boundary}$ are found using (4), by minimizing

$$D_{X-B}(b_B) = \left\| \delta_{boundary} - (\overline{\delta}_{boundary} + \Phi_{B-boundary} b_B) \right\|^2 \tag{8}$$

and calculating

$$\delta_{B-non-boundary} = \overline{\delta}_{B-non-boundary} + \Phi_{B-non-boundary} b_B \tag{9}$$

where $\Phi_{B-boundary}$ and $\Phi_{B-non-boundary}$ are the first and second parts of $\Phi_B$ corresponding to the boundary and non-boundary points respectively, $\overline{\delta}_{boundary}$ and $\overline{\delta}_{B-non-boundary}$ are the first and second parts of $\overline{X}_B$ corresponding to the boundary and non-boundary points respectively, and $b_B$ is the variation parameter estimated in (8) and taken into (9) to compute $\delta_{B-non-boundary}$.

# 4 Experiments and Results

## 4.1 Variations of Prediction accuracy of Br-SDM with different training samples

Fig.5 shows the prediction differences of the selected 82 points of a Br-SDM trained with 244 samples and 334 samples respectively. Compare with Fig.4, which shows the predictions differences of a Br-SDM trained only with 128 samples, we can see that the prediction differences reduce when the number of training samples increases. In case of a Br-SDM trained with 244 samples, the major differences stay below 1.0mm, with the mean 0.64mm; and in case of a Br-SDM trained with 334 samples, the major differences even stay below 0.5mm, however, the deviation is larger, with errors of some points are larger than 1.5mm or even up to 2.5mm. We interpret this observation as the result of over-training.

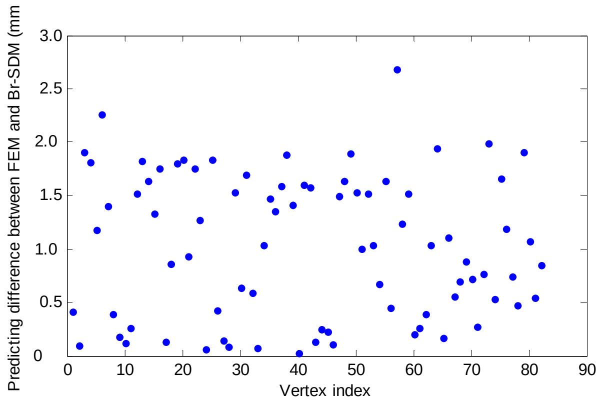



**Fig. 4.** Prediction differences of the selected 82 points from the original 128 samples ( in the case that the jawbone piece is moved forward 5.1mm)

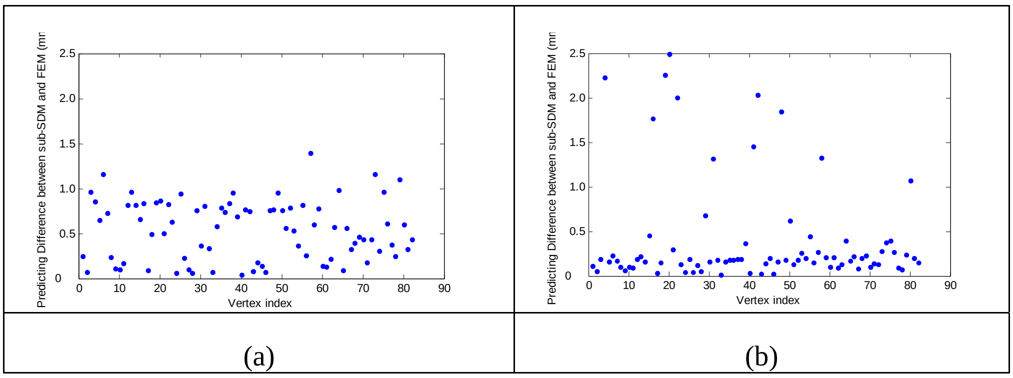



**Fig. 5.** Predicting differences of the selected 82 points (in the case that the jawbone piece is moved forward 5.1mm). (a) the prediction difference from the SDM trained with 244 samples, (b) the prediction difference for the SDM trained with 334 samples

When we apply the proposed mix-resolution Br-SDM that consists of the sub-SDM of the main deformation area and the main-SDM of the other facial region, we find that the average differences are significantly reduced (table 1).

**Table 1.** Mean predicting difference for different surgical plans.

| Surgical Plan | Mean Difference (Br-SDM) | Mean Difference (mBr-SDM) | Mean Difference (A) |
|---|---|---|---|
| F+3.1mm | 0.32mm | 0.12mm | 0.45mm |
| F+4.3mm | 0.44mm | 0.20mm | 0.58mm |
| F+5.1mm | 0.51mm | 0.32mm | 0.64mm |

*F+ is for Mandible Advancement. The mean difference is defined as $E = \frac{1}{n}\sum_{i=1}^{n} e(i)$. It can seen from Table 1 that the mean differences for mBr-SDM is significantly reduced compared with those for Br-SDM. A denotes the mean difference for the main deforming region calculated by the Mixed-resolution Br-SDM.

### 4.2 Computational Cost

By incorporating sub-SDM in the mixed-resolutions Br-SDM does not require much additional computational cost. The only additional cost is consumed in the process of training the sub-SDM. Since the sub-SDM typically covers only a small region of the face where the major deformation occurs, it typically consists a small number of vertices, (82 points in our example), the training of the high-resolution sub-SDM can be completed in our experiment in 1 minute using a PC with Intel Pentium M processor and 2Gbyte RAM with a matlab program.

## 5 Conclusion

In this paper, we proposed a Mixed-resolution Br-SDM (mBr-SDM) to improve the prediction accuracy of bone-related soft-tissue changes in orthognathic surgical planning while maintaining a low computational costs compared with FEM. Specifically, Mixed-resolution Br-SDM consists of a sub-SDM which serves to capture the detail deformation variations of the points around the main deforming areas of interest, while a low-resolution main-SDM is used to capture the deforming variations of other points of the facial regions. This way, we are able to focus the computations of detail deformation modes using more samples for the regions of interests within a SDM, while keeping the computational costs down compared with FEM.

## Reference


[1] Erwin Keeve, Sabine Girod, et.al. *Deformable Modeling of Facial Tissue for Craniofacial Surgery Simulation.* Computer Aided Surgery vol.3, pp:228–238 (1998)

[2] M Bro-Nielsen, HTM Inc, MD Rockville. *Finite Element Modeling in Surgery Simulation.* Proceedings of the IEEE, vol.86(33), pp:490-503 (1998)

[3] W.Mollemans, F.Schutyser et. al. *Predicting soft-tissue deformations for a maxillofacial surgery planning system: From computational strategies to a complete clinical validation*. Medical Image Analysis, vol.11(3), pp:282-301 (2007)

[4] Roose, L., De Maerteleire, W.,Mollemans, W.,Suetens,P. *Validation of different soft tissue simulation methods for breast augmentation*. Proceedings of CARS, pp:485-490 (2005)

[5] S Cotin, et.al .*A hybrid elastic model allowing real-time cutting, deformations and force-feedback for surgery training and simulation*. The Visual Computer, 16(88):437-452 (2000)

[6] T.F. Cootes, C.J.Taylor,D.H.Cooper,J.Graham. *Active Shape Models – Their Training and Application*. Computer Vision and Image Understanding, Vol.61(1), pp: 38-59 (1995)

[7] Charles Kervrann and Fabrice Heitz. *Statistical Deformable Model-Based Segmentation of Image Motion*. IEEE Transactions on Image Processing. Vol.8(4),pp:583-588(1999)

[8] S Meller, E Nkenke, W Kalender. *Statistical Face Models for the Prediction of Soft-tissue Deformations after Orthognathic Osteotomies*. MICCAI, vol. 3750, pp:443-450 (2005)

[9] S.J. Raudys, A.K. Jain. *Small Sample Size Effects in Statistical Pattern Recognition: Recommendations for Practitioners*. IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 13(3), pp. 252-264 (1991)

[10] Qizhen He, Jun Feng, Horace H. S. Ip , James Xia and Xianbin Cao. *An Integration of Statistical Deformable Model and Finite Element Method for Bone-related Soft Tissue Prediction in Orthognathic Surgery Planning*. Accepted by: 4th international workshop on Medical Imaging and Augmented Reality. Tokyo, August, 2008.