FFT based convolution

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Abstract

The Fourier transform of the convolution of two images is equal to the product of their Fourier transform. With this definition, it is possible to create a convolution filter based on the Fast Fourier Transform (FFT). The interesting complexity characteristics of this transform gives a very efficient convolution filter for large kernel images.

This paper provides such a filter, as well as a detailed description of the implementation choices and a performance comparison with the ”simple” itk::ConvolutionImageFilter.

Contents

1 Introduction

The convolution of two image is a very cpu demanding task, with a complexity of $O(N \times M)$ where N is the number of pixel of the first image and M the number of pixels of the second. Usually, the size of one of the image is a lot smaller than the one of the other – the smaller is often called the kernel. The quite small size of the kernel makes the convolution computable in a very usable time, but when the size of the kernel grows, the computation time quickly becomes impraticable.

Fortunately, the convolution of two images is simply the product of those two images pixel wise in the frequency domain, with a complexity of $O(max(N,M))$. The size of the kernel, which is usually way smaller than the image, has no effect on the computation time, so the complexity is simply $O(N)$. The cost of the Fourier Transform can be quite high however – that's why the FFT based convolution is more efficient only for big kernel images.

2 FFT based convolution step by step

The FFT based convolution requires several step to be performed. They are detailed one by one.

2.1 Padding

The image to convolve and the kernel will be padded with for several reasons:

- to make their size match. This is required to perform the multiplication in the frequency domain.
- to avoid the border effects. Because the FFT considere the image as a cyclic signal, the image must be padded to avoid the border effects. The final image size for all the dimensions must be at least $P+Q-1$, where P is the size of the first image on a dimension and Q the size of the other image on the same dimension.

- to make the FFT possible, or to enhance its performance. Some FFT implementations, like the VNL one, can only be run on an image where the size on all the dimensions are a power of two. Some other implementations, like FFTW's one, are performing very differently depending on the size of the image. Adding a few more pixels on the border of the image can lead them to perform a lot better, as shown in chapter 4.2.

The kernel is always padded with zeros, but the image to convolve can be padded in several ways:

- zeros – while simple, this method may show darkened border in the convolved image;
- zero flux Neumann – this method extends the pixels on the image border in the padded zone, and is quite efficient in keeping the border effect low;
- mirror – the image is padded with a mirror of the image. This is also a quite good way to keep the border effect low;
- wrap – the opposite border of the image is copied in the padded zone. This may be useful to keep the circular behavior of the FFT, while still padding to fit the size requirement of the FFT implementation.

The result of those padding can be seen in figure 1.

2.2 Normalization

To preserve the intensity of the pixel in the convolved image, the kernel must be normalized to one – the sum of all its pixel is one.

2.3 Flipping

The convolution is a flipping transform by nature: the shape of the kernel image can be found in the convolved image, but flipped on all the axes. To get the same behavior when computing the product in the frequency domain, the kernel image must be flipped prior to the FFT. See figure 2.

2.4 Shifting (centering)

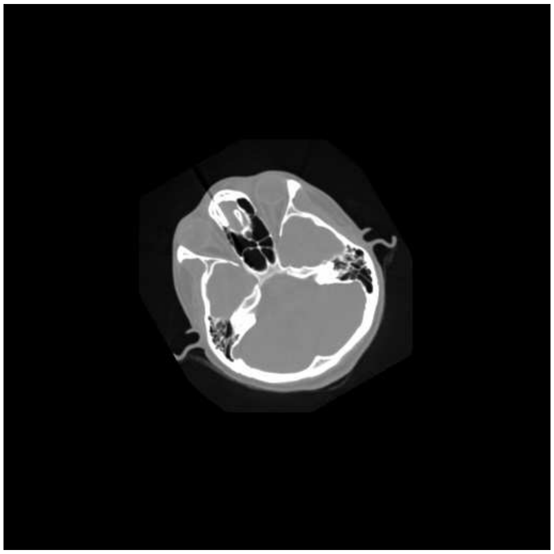
The convolution is usually done with a centered kernel. If the kernel is not centered, the input image is shifted in the convolved image, by the same shift than the shift of the kernel. However, for the FFT, the center pixel is the one in the corner, not the one in the center of the image, as is it usual for a human. To avoid a shift in the convolved image, the pixels must be shifted to the corner of the image as expeced by the FFT. See figure 2.

This shift must be done after the padding, and before the FFT.

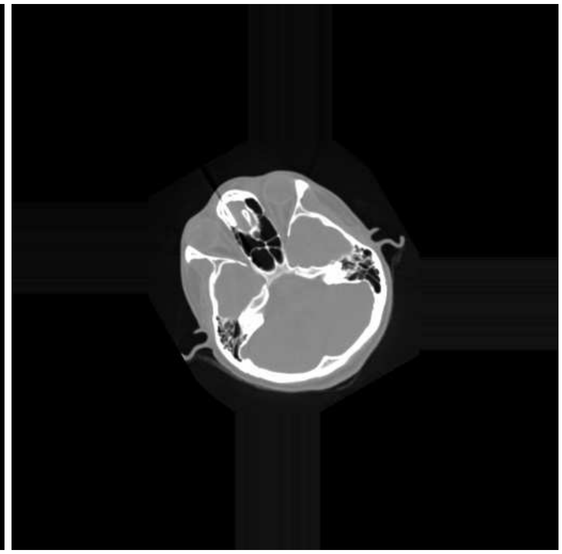
Another option, not used here, is to shift the image after the inverse Fourier transform.

2.5 Fourier transform

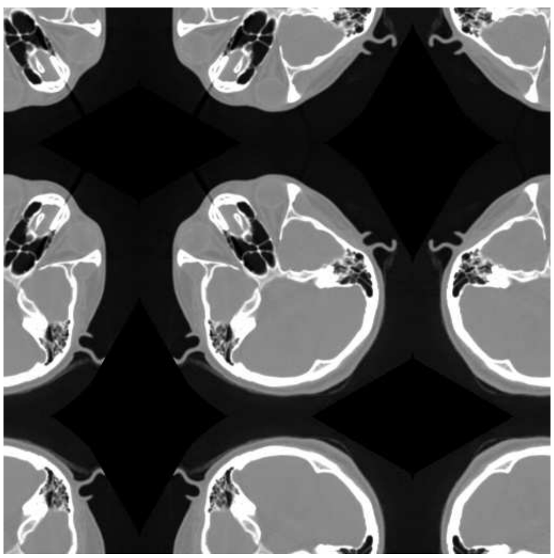
The Fourier transform is performed on the padded input image, and on the normalized, padded, flipped and shifted kernel.



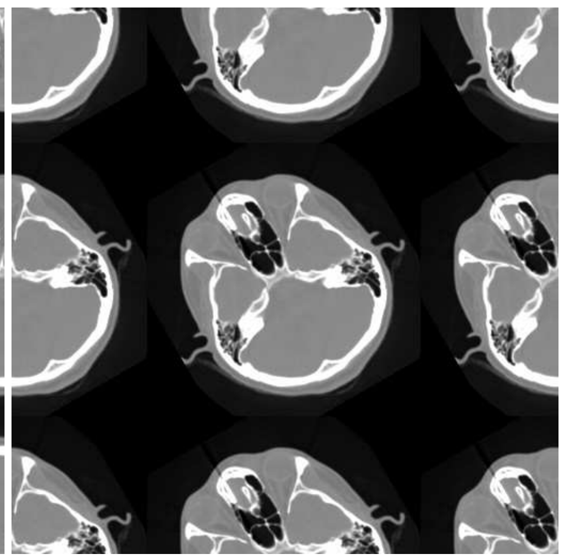
(a) Zero



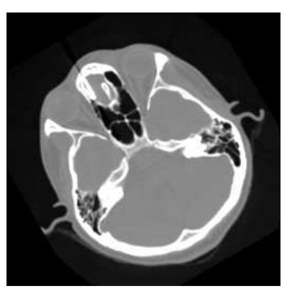
(b) Zero flux Neumann



(c) Mirror

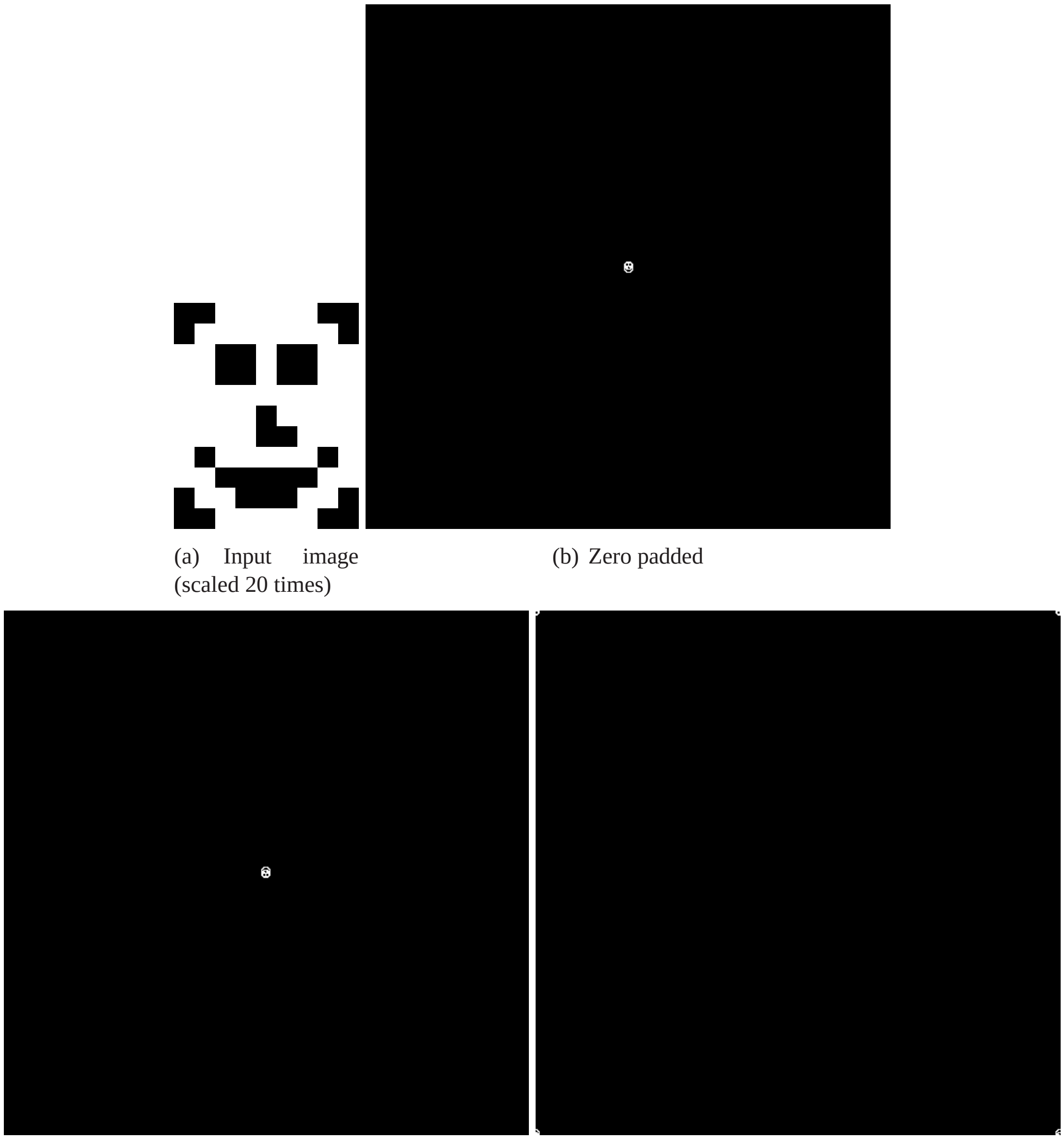


(d) Wrap



(e) Input image

Figure 1: Padding a 256 $\times$ 256 image to a 512 $\times$ 512 one in several ways.



(a) Input image (scaled 20 times)

(b) Zero padded

(c) Zero padded and flipped

(d) Zero padded, flipped and shifted

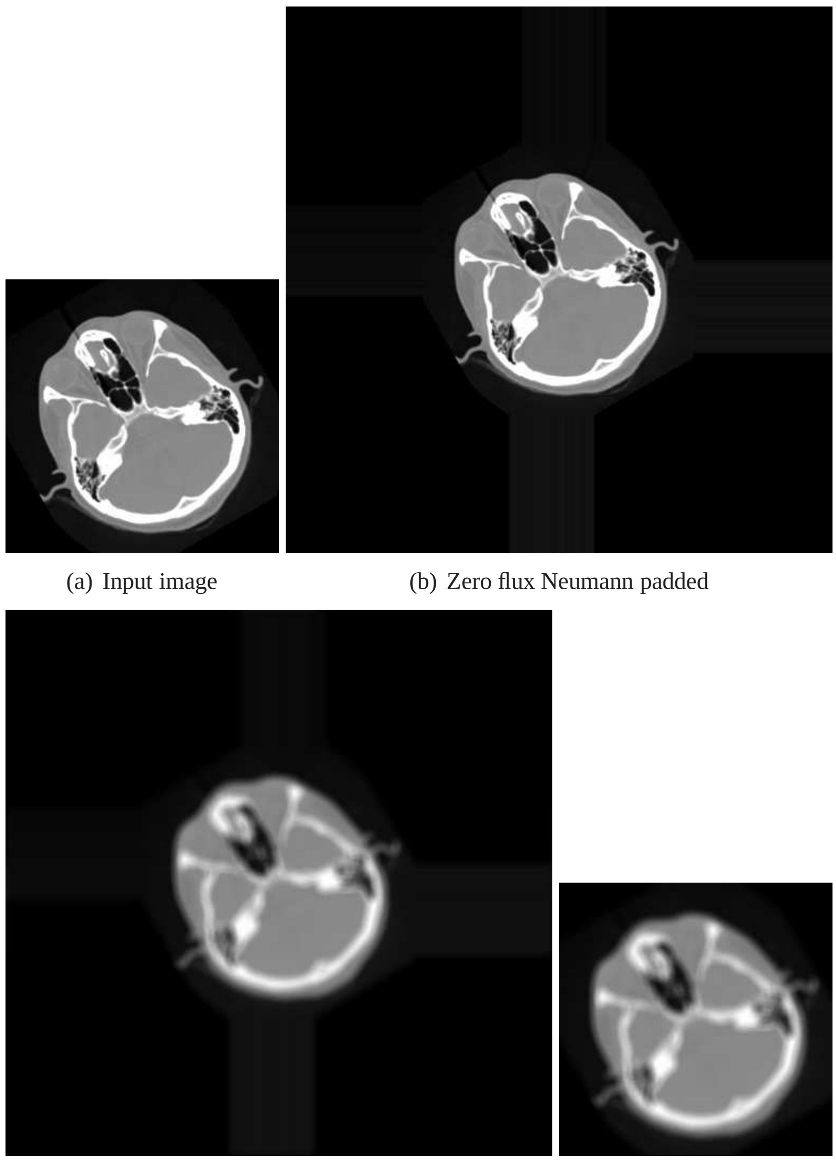
Figure 2: Kernel modifications in order to perform the FFT based convolution. Normalization is only a change in intensity, and thus is not shown here.

2.6 Frequency domain multiplication

The convolution is performed in the frequency domain, simply by computing the product of the FFT of the image and of the FFT on the kernel pixel wise.

2.7 Inverse Fourier transform

The convolved image in the frequency domain is transformed to the space domain with an inverse Fourier transform. See figure 3.



(a) Input image (b) Zero flux Neumann padded

(c) Convolved, still padded (d) Cropped to the input image size

Figure 3: The modifications applied to the input image.

2.8 Cropping

The image is cropped to fit the size of the input image. See figure 3.

3 Implementation

The step by step transform described in the previous chapter can perfectly be implemented in a pure ITK pipeline model. The few filters which were missing for this task have been implemented, and some other have been enhanced to improve their performance.

3.1 NormalizeToConstantImageFilter

The normalization to one cannot be done without going outside of the ITK pipeline model at this time. itk::NormalizeToConstantImageFilter has been added for this task.

This filter is implemented as a minipipeline of two filters:

- a itk::StatisticsImageFilter, to compute the sum of the pixels in the image;
- a itk::DivideByConstantImageFilter, to actually normalize the pixel values.

The use of itk::StatisticsImageFilter only to compute the sum might be *a little overkill*. This filter is only used on the kernel, which is usually quite small, so the performance impact on the whole FFT based convolution is small.

3.2 ZeroFluxNeumannPadImageFilter

The zero flux Neumann padding filter was not implemented in ITK. It has been implemented by modifying the `ConstantPadImageFilter` code.

3.3 FFTPadImageFilter

The padding step, as described in the previous chapter, has several goals, which lead to a single image size used to pad both the input image and the kernel. All the logic is implemented in `itk::FFTPadImageFilter`.

This filter is in charge of padding both images. It requires two input – the image to convolve and the kernel. The pad size is computed according to both image sizes.

The image to convolve is simply padded with one of the available method: zeros, zero flux Neumann, mirror or wrap.

The kernel, however, requires a little more work to be centered in the padded image, and to make its region match the one of the padded image to convolve. The padding and the region changes are implemented as a minipipeline of `itk::ConstantPadImageFilter` and `itk::ChangeInformationImageFilter`.

`itk::FFTPadImageFilter` is also able to extend the padded region in order to enhance the FFT performance, or to make it possible when the FFT possible when it require a size which is a power of two. FFTW is able to work on any image size, but due to the algorithms useds, produce significantly more performant result with some specific sizes. The two main reasons are:

- the decomposition of images of composite sizes in smaller transforms using the Cooley-Tukey algorithm;
- the hard coded loop unrolling for the size which are prime numbers up to thirteen.

For these reasons, FFTW performs better when the size on each dimension has its greatest prime factor smaller or equal to thirteen. The greatest prime factor can be simply computed with the algorithm described in algorithm 3.2.

Algorithm 3.1: ISPRIME(n)

for each $x \in [2, \sqrt{n}]$
  do $\begin{cases} \textbf{if } n\%x = 0 \\ \quad \textbf{then return } (\textbf{ false }) \end{cases}$
return (**true**)

Algorithm 3.2: GREATESTPRIMEFACTOR(n)

$x \leftarrow 2$
while $x \leq n$
  do $\begin{cases} \textbf{if } n \bmod x = 0 \textbf{ and } \textsc{isPrime}(x) \\ \quad \textbf{then } n \leftarrow n/x \\ \quad \textbf{else } v \leftarrow v + 1 \end{cases}$
return (x)

`FFTPadImageFilter` can also be used with a single input. This is useful to be able to run a FFT on an image with a size which is not a power of 2 with the vnl implementation, or to enhance the performance of the FFTW implementation, without necessairly doing a convolution.

3.4 FFTW filters enhancements

ITK provides FFT and inverse FFT filters based on the FFTW library. Those filters had some problems, or were missing some features. They have been enhanced to fit the needs of the FFT based convolution.

Thread support

The FFTW filters were not multithreaded. There are two places where multithreading can take place: the FFTW execution and the normalization step.

The FFTW execution can be threaded simply by calling `fftwf_plan_with_nthreads(threads);` or `fftwf_plan_with_nthreads(threads);`, depending on the pixel type, before creating the plan. `itk::fftw::Proxy` has been modified to provide this feature: an extra `thread` parameter, which defaults to 1 to keep the backward compatibility, has been added to the method in charge of creating the plans. Then the `itk::FFTWComplexConjugateToRealImageFilter` and `itk::FFTWRealToComplexConjugateImageFilter` have been modified to pass their user defined number of threads to those methods.

In order to implement the threading support in the normalization step in `itk::FFTWComplexConjugateToRealImageFilter`, the execution of the FFTW library has been moved to `BeforeThreadedGenerateData()`, and the normalization step to `ThreadedGenerateData()`.

Memory usage reduction

FFTW filters were using a lot of memory, because they were allocating an internal buffer for the input image, and an other one for the output image. Worth, these buffers were not deallocated at the end of the execution filter, keeping the memory usage very high even when the execution has completed.

I guess the main reasons to do that were:

- avoid the destruction of the input. The creation of the FFTW plan, and the execution of FFTW can destroy the input. This is most of the time not acceptable in ITK.
- reuse the FFTW plan for a latter run. The plan include the memory location of the input and output buffers, so the buffers can't be deallocated, as it may be the case with the filter's input and output images.

The destruction of the input can be avoided in the creation plan by using the `FFTW_ESTIMATE` flag – that flag was already used in the implementation. It can also be avoided while running FFTW, for the real to complex transform, by also using the flag `FFTW_PRESERVE_INPUT`. So in `itk::FFTWRealToComplexConjugateImageFilter`, if the right flags are used, there is no need for the input buffer. The case is a bit more difficult for the complex to real transform: FFTW does not provide any algorithm able to preserve the input, so the input must be copied to an internal buffer. There is a case however, when we don't care about destroying the input in ITK: when the `ReleaseDataFlag` is `ON` on the input of the filter. In that case, the input will be deallocated right after the end of the execution of the filter, and so there is no need to take care of the input – it can be destroyed by the filter, and then deallocated. `itk::FFTWRealToComplexConjugateImageFilter` has been modified to use the input image directly without intermediate buffer when `ReleaseDataFlag` is `ON`. The detection of the `ReleaseDataFlag` is a bit difficult during the pipeline execution: this flag is always modified to `OFF` before the call to `GenerateData()` to avoid the destruction by a minipipeline. In consequence, the detection of the `ReleaseDataFlag` is done at an earlier stage in the pipeline execution, in `UpdateOutputData()`, and stored in a member variable: `m_CanUseDestructiveAlgorithm`.

The later reuse of the plan, while sensible when not using the `FFTW_ESTIMATE` flag, is not much interesting when it is used: the time needed to create the plan is very short – way shorter than the FFT computation. Also, the number of threads and the image size are stored in the plan, and can't be changed later, which doesn't fit well with ITK pipeline model. In the enhanced version, the plan is not kept between the filters execution, and so no intermediate output buffer are used, decreasing significantly the needed memory.

Another reason to create the internal buffer may be the need to align properly the image, to significantly improve the FFTW performance by using the SIMD operations. The buffer were simply allocated with the `new` operator, which is not enough to ensure a proper alignement on some plateforms, including Microsoft Windows and Linux.

Code safety

Only `fftwf_execute(plan);` and `fftw_execute(plan);` are thread safe in FFTW – the other functions are not thread safe. In consequence, the call to all the other functions has been protected by a global lock in the enhanced classes.

Also, the check for the size of the image to know if the plan can be reused may not have been good enough, because it was testing only the number of pixels and not the size on all the dimensions. The plan is not kept between the execution, so this can not be a problem anymore.

3.5 RegionFromReferenceImageFilter

At the end of the convolution, the image is cropped to return to the size of the input image. Again there is currently no option in ITK to do that fully in the pipeline model. In consequence, a new filter has been developped for this task.

`itk::RegionFromReferenceImageFilter` is implemented as a subclass of `itk::ExtractImageFilter`. This filter requires two inputs – the first is the image to crop, the second is simply used to get the region to extract in the first one. The reference image – the second input – is simply expected to be of type `itk::ImageBase` with the same dimension than the first one, and so it's type don't need to be specified in the template parameters.

3.6 FFTConvolutionImageFilter

`itk::FFTConvolutionImageFilter` is a convenient minipipeline filter which groups all the required filters to make the FFT based convolution in a single and easy to use filter. See figure 4 for an overview of the minipipeline.

It sets the `ReleaseDataFlag` properly to keep the memory usage as low as possible, and propagate the user specified number of threads.

It also expose the `GreatestPrimeFactor` accessors from `itk::FFTPadImageFilter` to let the user choose between the extra performance added by the padding and the extra memory cost, and `PadMethod` from the same filter to let the user choose the padding method – zero flux Neumann is used by default.

Finally, the `Normalize` attribute let the user choose if the kernel must be normalized to one or not.

4 Performance

All the performance tests are run on a workstation with two Intel(R) Xeon(R) CPU X5570 at 2.93GHz and 24 GB of RAM and running Mandriva Linux 2009.1.

4.1 Kernel size

As visible in figure 7, the kernel size as a little effect on the execution time for the FFT based convolution, but a quite big impact on the normal one.

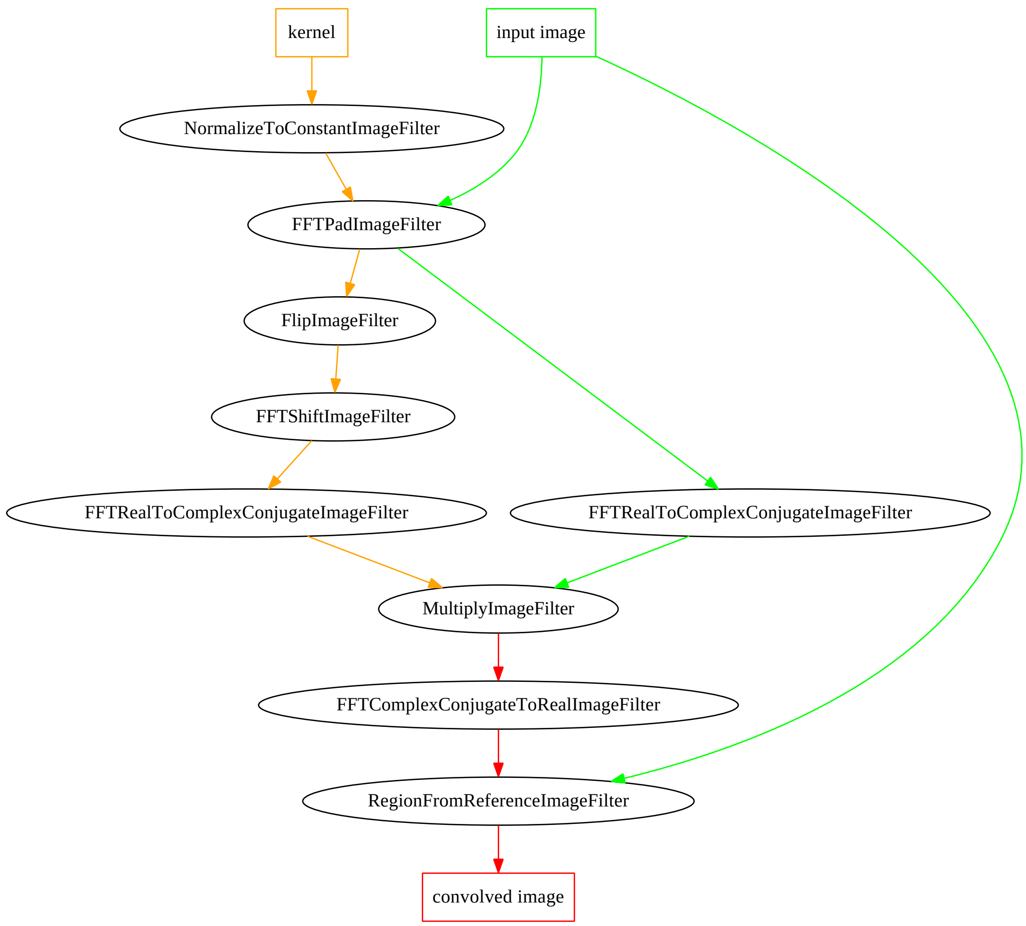


Figure 4: Internal pipeline of FFTConvolutionImageFilter.

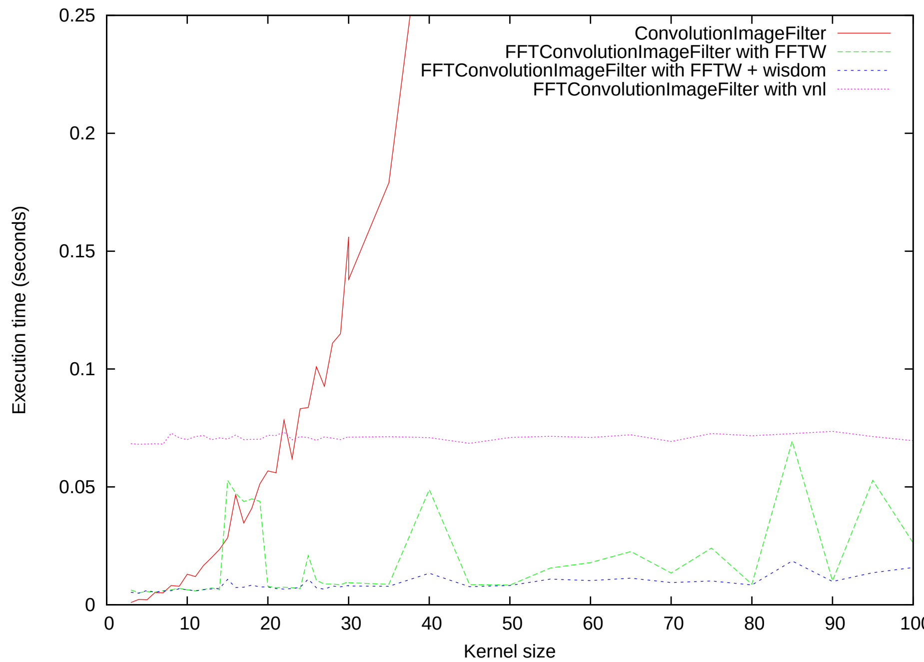


Figure 5: The effect of the kernel size on the execution time. The input image size is 256×256.

The drop in performance at some times seems to be due to a not so good plan generated by FFTW with `FFTW_ESTIMATE`. Using FFTW's wisdom feature can almost remove that problem. An integration of wisdom in ITK's FFTW filters will be proposed in a later contribution.

4.2 Greatest prime factor

As visible in figure 6, the FFTW library performs quite differently depending on the size of the input. A good way to get the best performance seems to be to use the sizes with the greatest prime factor smaller or equal to 13.

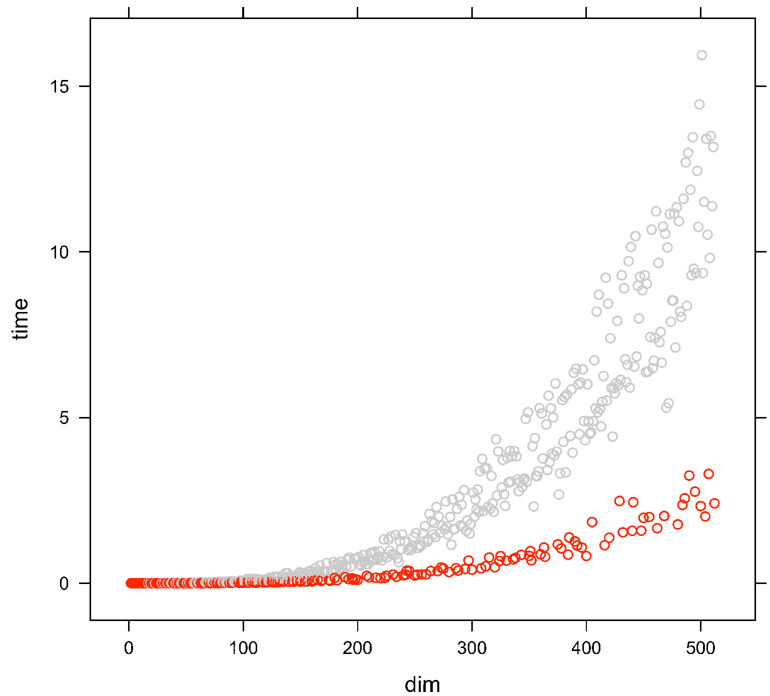
Most of the time, and despite the usual advice to use a size which is a power of two, those sizes perform significantly better than the closest sizes which are a power of two, probably because the performance gain per pixel doesn't compensate the higher number of pixels in the image.

4.3 Number of threads

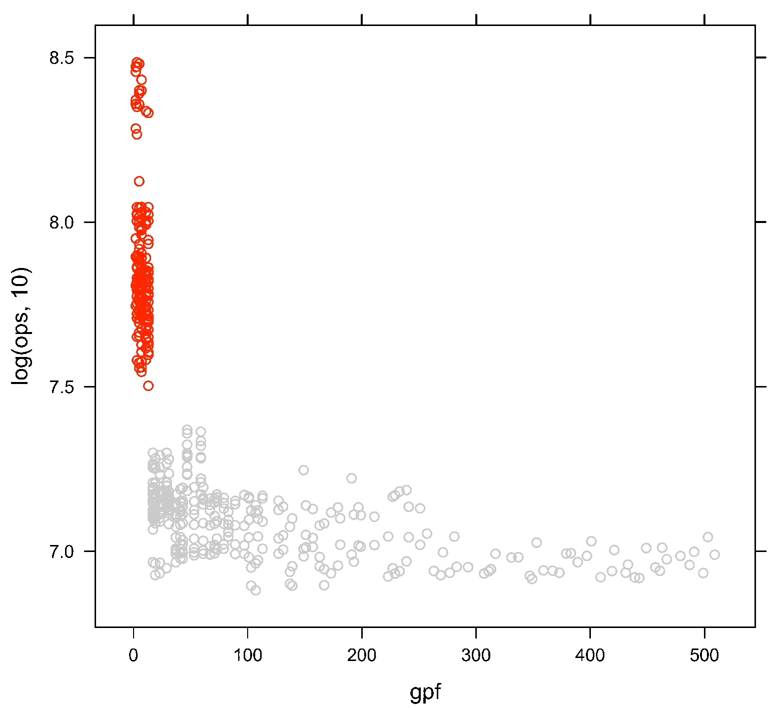
As visible in figure 7, the FFT based convolution and the normal one are scaling quite well with the number of threads.

5 Wrapping

All the new filters have been wrapped using WrapITK.



(a) Execution time for an image of size dim^3.



(b) Effect of the image greatest prime factor of the image size (gpf) on the number of pixels processed per seconds (ops).

Figure 6: Effect of the image size on the FFTW performance. Red dots are the sizes with a greatest prime factor smaller or equal to 13. The grey dots are the ones with a greatest prime factor greater than 13.

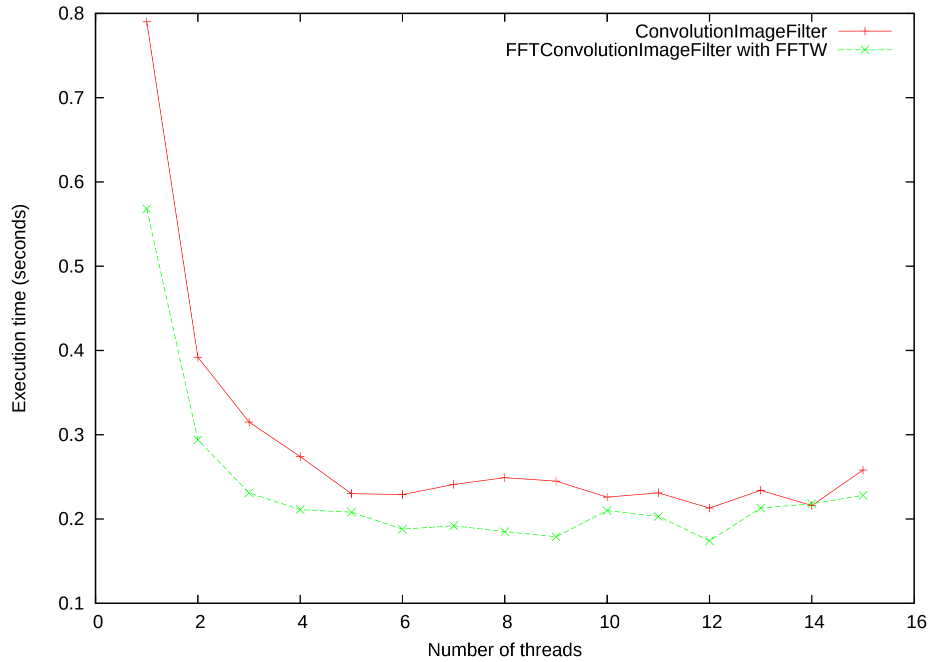


Figure 7: The effect of the number of threads on the execution time. The input image size is $371 \times 371 \times 34$, and the kernel size is two.

6 Development version

A development version is available in a darcs repository at http://mima2.jouy.inra.fr/darcs/contrib-itk/fftco

7 Conclusion

This contribution provides with an efficient implementation of the convolution transform for large kernels. The FFTW filters implementations has been significantly enhanced, and a set of reusable tools is provided.

Some other work may be done on this basis – an non exhaustive list may be:

- a binary erosion/dilation based on FFT;
- a set of deconvolution filters;
- a more memory efficient implementation of the convolution filter using the Overlap-Save algorithm;
- let the user create a plan with `FFTW_PATIENT`, `FFTW_MEASURE`, or `FFTW_EXHAUSTIVE`, so that FFTW can create better plan with `FFTW_ESTIMATE` in a later run;
- integrate FFTW's wisdom to improve FFT performance in ITK.

References

[1] L. Ibanez and W. Schroeder. *The ITK Software Guide*. Kitware, Inc. ISBN 1-930934-10-6, http://www.itk.org/ItkSoftwareGuide.pdf, 2003.